On the rank of pseudo walk matrices

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Abstract

In the literature, the walk matrix $W_b$ associated with a graph $G$ having vertex set $V(G)$ is the matrix with columns $b, A b, A^2 b, \ldots, A^{r-1} b$ that enumerates the number of all possible walks on $G$ of length $0, 1, 2, \ldots, r-1$ starting from each vertex of $G$ and ending at any of the vertices indicated by $b$. We generalize walk matrices further to obtain pseudo walk matrices $W_v$ having any walk vector $v$. For any subset $S$ of $V(G) \times V(G)$, the total number of walks $N_0(S), N_1(S), N_2(S), \ldots$ of length $0, 1, 2, \ldots$ in $G$ that start from vertex $i$ and end at vertex $j$ for all $(i, j) \in S$ is considered. Various results on such pseudo walk matrices are presented, particularly related to their rank. The matrix rank of pseudo walk matrices allows the consideration of controllable and recalcitrant pairs.