Quantum polynomial optimisation problems for dimension $d$ variables, with symmetries

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Quantum Information Theory (QIT) involves quantum states and measurements, mathematically represented as non-commutative positive operators. A typical problem in QIT is to find the minimum of a polynomial expression in these operators. For instance, finding the maximum violation of a Bell inequality, or the ground state energy of a Hamiltonian are polynomial optimisation problems in non-commuting variables. The NPA hierarchy [New J. Phys. 10, 073013 (2008)], which can be viewed as the “eigenvalue” version of Lasserre’s hierarchy, provides a converging hierarchy of SDP relaxation of a non-commutative polynomial optimisation problem involving variables of unbounded dimension. This hierarchy converges, it is one of QIT main technical tool.

Importantly, some QIT problems concern operators of bounded dimension $d$. The NPA hierarchy was extended into the NV hierarchy [Phys. Rev. Lett. 115, 020501(2015)] to tackle this case. In this method, one first sample at random many dimension $d$ operators satisfying the constraint, and compute the associated moment matrix. This first step discovers the moment matrix vector space, over which the relaxed SDP problem is solved in a second step. In this talk, we will first review this method. Then, based on [Phys. Rev. Lett. 122, 070501], we will show how one can reduce the computational requirements by several orders of magnitude, exploiting the eventual symmetries present in the optimization problem.