Regular 1-factorizations of complete graphs with orthogonal spanning trees

Gloria Rinaldi
University of Modena and Reggio Emilia
gloria.rinaldi@unimore.it

A 1-factorization $\mathcal{F}$ of a complete graph K_{2n} is said to be G-regular if G is an automorphism group of $\mathcal{F}$ acting sharply transitively on the vertex-set. The problem of determining which groups can realize such a situation dates back to a result by Hartman and Rosa (1985) which solved the problem when G is a cyclic group. It is also well known that this problem simplifies somewhat when n is odd: G must be the semi-direct product of Z_2 with its normal complement and G always realizes a $1-$factorization of K_{2n} upon which it acts sharply transitively on vertices. When n is even the problem is still open, even though several classes of groups were tested in the recent past. An attempt to obtain a fairly precise description of groups and 1-factorizations satisfying this symmetry constraint could be done by imposing further conditions. For example some non existence results were achieved by assuming the existence of a 1-factor fixed by the action of the group, further results were obtained when the number of fixed $1-$factors is as large as possible. In this talk we focus our attention on the possibility of constructing G-regular 1-factorizations of K_{2n} together with a complete set of isomorphic spanning trees orthogonal to the $1-$factorization. Here orthogonal tree means that the tree shares exactly one edge with each $1-$factor. We see how to realize such a situation when n is odd and examine some classes of groups in the case n even.