Factorizations of infinite graphs

Simone Costa  
*University of Brescia*  
simone.costa@unibs.it  

Tommaso Traetta  
*University of Brescia*  
tommaso.traetta@unibs.it

Let $\mathcal{F} = \{F_\alpha : \alpha \in \mathcal{A}\}$ be a family of infinite graphs. The Factorization Problem $FP(\mathcal{F}, \Lambda)$ asks whether $\mathcal{F}$ can be realized as a factorization of a given infinite graph $\Lambda$, namely, whether there is a factorization $\mathcal{G} = \{\Gamma_\alpha : \alpha \in \mathcal{A}\}$ of $\Lambda$ such that each $\Gamma_\alpha$ is a copy of $F_\alpha$.

Inspired by the results on regular 1-factorizations of infinite complete graphs [1] and on the resolvability of infinite designs [4], we study this problem when $\Lambda$ is either the Rado graph $R$ or the complete graph $K_\aleph$ of infinite order $\aleph$. When $\mathcal{F}$ is a countable family, we show that $FP(\mathcal{F}, R)$ is solvable if and only if each graph in $\mathcal{F}$ has no finite dominating set. Generalizing the existence result of [2], we also prove that $FP(\mathcal{F}, K_\aleph)$ admits a solution whenever the cardinality $\mathcal{F}$ coincides with the order and the domination numbers of its graphs.

Finally, in the case of countable complete graphs, we show some non-existence results when the domination numbers of the graphs in $\mathcal{F}$ are finite.

References


