The modularity conjecture for elliptic curves is most famous in its formulation for elliptic curves over the rational numbers – indeed, Wiles proved the modularity of semistable elliptic curves over the rationals as part of his proof of Fermat’s Last Theorem.

Recently it has become possible to attack the problem of modularity of elliptic curves over more general number fields. I will explain what this means, what we know, and what kinds of consequences we can expect for the solution of Diophantine equations.