On Pleijel's nodal domain theorem for quantum graphs

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In this talk we present recent results on metric graph counterparts of Pleijel's theorem on the asymptotics of the number of nodal domains ν_n of the n-th eigenfunction(s) of a broad class of operators on compact metric graphs, including Schrödinger operators with L^1-potentials as well as the p-Laplacian with natural vertex conditions, and without any assumptions on the lengths of the edges, the topology of the graph, or the behaviour of the eigenfunctions at the vertices. We characterise the accumulation points of the sequence $(\frac{\nu_n}{n})_{n\in\mathbb{N}}$, which are shown to form a finite subset of $(0,1]$. This extends the previously known result that $\nu_n \sim n$ *generically*, for certain realisations of the Laplacian, in several directions. In particular, we will see that the existence of accumulation smaller than 1 is strictly related to the failure of the unique continuation principle on metric graphs.

Finally we show that for (most) metric graphs – metric trees and general metric graphs with at least one Dirichlet vertex – there exists an infinite sequence of *generic* eigenfunctions – namely, eigenfunctions of multiplicity 1 that do not vanish in the graph's vertices – of the free Laplacian and infer that, in this case, 1 is always an accumulation point of $\frac{\nu_n}{n}$. In order to do so we introduce a new type of secular function.

The talk is based on joint work with Matthias Hofmann (Lisbon), James Kennedy (Lisbon), Delio Mugnolo (Hagen) and Matthias Täufer (Hagen).