Surjective isometries on Banach algebras of Lipschitz maps taking values in a unital $C^*$-algebra

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Let $\mathcal{A}$ be a unital $C^*$-algebra. If its center is trivial, i.e. $\mathcal{A} \cap \mathcal{A}' = \mathbb{C}1$, we call it a unital factor $C^*$-algebra. In this talk, we consider unital surjective complex linear isometries on $\text{Lip}(X, \mathcal{A})$ with $\| \cdot \|_L = \| \cdot \|_\infty + L(\cdot)$. The celebrated Kadison theorem yields that unital surjective linear isometries between unital $C^*$-algebras are Jordan $\ast$-isomorphisms. We consider surjective linear isometries on Banach algebras of continuous maps taking values in a $C^*$-algebra and derive extensions of the Kadison theorem.

In [O. Hatori, K. Kawamura and S. Oi, Hermitian operators and isometries on injective tensor products of uniform algebras and $C^*$-algebras, JMAA, 2019], we proved that if $\mathcal{A}_i$ is a unital factor $C^*$-algebra for $i = 1, 2$, every surjective linear isometry $U$ from $C(K_1, \mathcal{A}_1)$ onto $C(K_2, \mathcal{A}_2)$ is a weighted composition operator of the form $UF(y) = uV_y(F(\varphi(y)))$, where $\varphi : K_2 \to K_1$ is a homeomorphism, $\{V_y\}_{y \in K_2}$ is a strongly continuous family of Jordan $\ast$-isomorphisms from $\mathcal{A}_1$ onto $\mathcal{A}_2$, and $u \in C(K_2, \mathcal{A}_2)$ is a unitary element. A main result in this talk is a version of the Banach algebras of all Lipschitz maps of the theorem. Recently in [S. Oi, Hermitian operators and isometries on algebras of matrix-valued Lipschitz maps, Linear Multilinear Algebra, 2020], we gave a complete description of surjective linear isometries on $\text{Lip}(X, M_n(\mathbb{C}))$, where $M_n(\mathbb{C})$ is the Banach algebra of complex matrices of order $n$. Hence the main result is the generalization of it.

In the course of the proof, we characterize hermitian operators on $\text{Lip}(X, E)$ with $\| \cdot \|_L$ for any Banach space $E$. Note that similar results characterizing hermitian operators on $\text{Lip}(X, E)$ with $\| \cdot \|_M = \max\{\| \cdot \|_\infty, L(\cdot)\}$ have been already obtained in [F. Botelho, J. Jamison, A. Jiménez-Vargas and M. Villegas-Vallecillos, Hermitian operators on Lipschitz function spaces, Studia Math., 2013].

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