Converting divergent asymptotic series into convergent series: factorial series for Laplace-type integrals

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Asymptotic techniques for Laplace-type integrals are a useful tool to derive asymptotic approximations of special functions. But in most of the important examples of special functions, the asymptotic expansion is not convergent. In this paper we investigate a modification of those asymptotic techniques that transforms the unbounded integration region of the Laplace-type integral into a bounded region. Then, an elementary asymptotic analysis of the new integral shows that the asymptotically relevant integration point is attained at a point of the boundary of the integration region and an expansion of the integrand at that point gives an asymptotic expansion of the integral. But moreover, an analysis of the remainder of this new expansion shows that it is convergent under a mild condition over the integrand. We illustrate this modification with several examples of special functions, providing convergent and asymptotic expansions of these functions.