We study Hermitian manifolds with a compact Lie group action by holomorphic isometries with principal orbits of codimension one. In particular, we focus on a special class of these manifolds constructed by following Bérard-Bergery, which includes, among the others, the holomorphic line bundles on $\mathbb{C}P^{m-1}$, the linear Hopf manifolds and the Hirzebruch surfaces. We characterize their invariant special Hermitian metrics, such as balanced, Kähler-like, pluriclosed, locally conformally Kähler, Vaisman, Gauduchon. Furthermore, we construct new examples of cohomogeneity one Hermitian metrics solving the second-Chern-Einstein equation and the constant Chern-scalar curvature equation.