Stability and asymptotic properties of dissipative equations coupled with ordinary differential equations

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In this talk, we will present some stability results of a system corresponding to the coupling between a dissipative equation (set in an infinite dimensional space) and an ordinary differential equation. Namely we consider $U, P$ solution of the system

\[
\begin{cases}
    U_t = AU + MP, \quad \text{in } H, \\
    P_t = BP + NU, \quad \text{in } X, \\
    U(0) = U_0, P(0) = P_0,
\end{cases}
\]

where $A$ is the generator of a $C_0$ semigroup in the Hilbert space $H$, $B$ is a bounded operator from another Hilbert space $X$, and $M, N$ are supposed to be bounded operators. Many problems from physics enter in this framework, let us mention dispersive medium models, generalized telegraph equations, Volterra integro-differential equations, and cascades of ODE-hyperbolic systems. The goal is to find sufficient (and necessary) conditions on the involved operators $A, B, M$ and $N$ that guarantee stability properties of system (1), i.e., strong stability, exponential stability or polynomial one. We will illustrate our general results by an example of generalized telegraph equations set on networks.