In this lecture, I will present a collection of results concerning the interplay between the scalar curvature of a Riemannian manifold and the mean curvature of its boundary, with special emphasis on dimension-dependent phenomena. Our work is motivated by a network of far-reaching conjectures by Gromov on the one hand, and by the study of the space of admissible initial data sets for the Einstein field equation in general relativity on the other.