

Zero sets of Laplace eigenfunctions

Alexandr Logunov

Princeton University

`alogunov@princeton.edu`

In the beginning of 19th century Napoleon set a prize for the best mathematical explanation of Chladni's resonance experiments. Nodal geometry studies the zeroes of solutions to elliptic differential equations such as the visible curves that appear in Chladni's nodal portraits. We will discuss the geometrical and analytic properties of zero sets of harmonic functions and eigenfunctions of the Laplace operator. For harmonic functions on the plane there is an interesting relation between local length of the zero set and the growth of harmonic functions. The larger the zero set is, the faster the growth of harmonic function should be and vice versa. Zero sets of Laplace eigenfunctions on surfaces are unions of smooth curves with equiangular intersections. Topology of the zero set could be quite complicated, but Yau conjectured that the total length of the zero set is comparable to the square root of the eigenvalue for all eigenfunctions. We will start with open questions about spherical harmonics and explain some methods to study nodal sets, which are zero sets of solutions of elliptic PDE.