Classes of strongly regular signed graphs

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We consider a concept of strong regularity defined for signed graphs – a generalization of strong regularity of the unsigned ones. We say that the signed graph $\hat{G}$ is strongly regular (for short, $\hat{G}$ is a SRSG) whenever it is regular, neither homogeneous complete nor totally disconnected, and if its adjacency matrix $A_{\hat{G}}$ satisfies

$$A_{\hat{G}}^2 = \frac{a}{2}(A_{\hat{G}} + A_G) - \frac{b}{2}(A_{\hat{G}} - A_G) + cA_G + rI,$$

where $G$ and $\overline{G}$ are the underlying graph of $\hat{G}$ and its complement and $r$ is the vertex degree of $G$ (and $\hat{G}$).

We establish certain basic structural and spectral properties of such signed graphs, and suggest a natural way to divide all SRSGs into five classes according to the relations among their defining parameters, which allows us to better perceive their properties.

Next, we consider walk-regularity of SRSGs with a relatively small number of distinct eigenvalues, belonging to some of those specified classes. In the end, we investigate the relationship between SRSGs with three or four distinct eigenvalues and three-class symmetric association schemes.