Observability for Non-Autonomous Systems

Fabian Gabel  
*Hamburg University of Technology*  
fabian.gabel@tuhh.de

Clemens Bombach  
*Technische Universität Chemnitz*  
clemens.bombach@mathematik.tu-chemnitz.de

Christian Seifert  
*Hamburg University of Technology*  
christian.seifert@tuhh.de

Martin Tautenhahn  
*Universität Leipzig*  
martin.tautenhahn@math.uni-leipzig.de

We study non-autonomous abstract Cauchy problems

\[ \dot{x}(t) = A(t)x(t), \quad y(t) = C(t)x(t), \quad t > 0, \quad x(0) = x_0 \in X, \]

where \( A(t) : D(A) \to X \) is a strongly measurable family of operators on a Banach space \( X \) and \( C(t) \in \mathcal{L}(X,Y) \) is a family of bounded observation operators from \( X \) to a Banach space \( Y \).

For measurable subsets \( E \subseteq (0,T), \quad T > 0 \), we provide sufficient conditions such that the Cauchy problem satisfies a final state observability estimate

\[
\|x(T)\|_X \lesssim \left( \int_E \|y(t)\|_Y^r \, dt \right)^{1/r}, \quad r \in [1, \infty),
\]

where an analogous estimate holds for the case \( r = \infty \).

An application of the above result to families of strongly elliptic differential operators \( A(t) \) and observation operators

\[ C(t)u := 1_{\Omega(t)}u, \quad \Omega(t) \subseteq \mathbb{R}^d, \quad u \in L^p(\mathbb{R}^d), \]

is presented. In this setting, we give sufficient and necessary geometric conditions on the family of sets \( (\Omega(t)) \) such that the corresponding Cauchy problem satisfies a final state observability estimate.