First order Mean Field Games on networks

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The theory of Mean Field Games studies the asymptotic behaviour of differential games (mainly in terms of their Nash equilibria) as the number of players tends to infinity. In these games, the players are rational and indistinguishable: each player aims at choosing its trajectory so to minimize a cost which depends on the trajectory itself and on the distribution of the whole population of agents. We focus our attention on deterministic Mean Field Games with finite horizon in which the states of the players are constrained in a network (in our setting, a network is given by a finite collection of vertices connected by continuous edges which cannot self-intersect). In these games, an agent can control its dynamics and has to pay a cost formed by a running cost depending on the evolution of the distribution of all agents and a terminal cost depending on the distribution of all agents at terminal time. As in the Lagrangian approach, we introduce a relaxed notion of Mean Field Games equilibria and we shall deal with probability measures on trajectories on the network instead of probability measures on the network. This is a joint work with: Y. Achdou (Univ. of Paris), P. Mannucci (Univ. of Padova) and N. Tchou (Univ. of Rennes).