Δ_\overline{\partial}-Harmonic forms on compact almost Hermitian manifolds

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Let $M$ be a smooth manifold of dimension $2n$ and let $J$ be an almost-complex structure on $M$. Then, $J$ induces on the space of forms $\mathcal{A}^\bullet(M)$ a natural bigrading, namely

$$\mathcal{A}^\bullet(M) = \bigoplus_{p+q=\bullet} A^{p,q}(M).$$

Accordingly, the exterior derivative $d$ splits into four operators

$$d : A^{p,q}(M) \to A^{p+2,q-1}(M) \oplus A^{p+1,q}(M) \oplus A^{p,q+1}(X) \oplus A^{p-1,q+2}(M)$$

$$d = \mu + \partial + \overline{\partial} + \overline{\mu},$$

where $\mu$ and $\overline{\mu}$ are differential operators that are linear over functions.

Let $g$ be a Hermitian metric on $(M,J)$. Denote by

$$\Delta_{\overline{\partial}} := \overline{\partial} \overline{\partial}^* + \overline{\partial}^* \overline{\partial}$$

the $\overline{\partial}$-Laplace. Then $\Delta_{\overline{\partial}}$ is an elliptic differential operator. We study the space of $\overline{\partial}$-harmonic forms on $(M,J,g)$. Special results are obtained for $\dim \mathbb{R} M = 4$. This a joint work with Nicoletta Tardini.