Gradient flow structure of a sixth order parabolic equation

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The second order linear heat equation and the fourth order nonlinear DLSS equation are gradient flows in the $L^2$-Wasserstein metric, for the entropy $H(\rho) = \int \rho \log \rho$ and the Fisher information $F(\rho) = \int \rho |\nabla \log \rho|^2$, respectively. Whereas $H$ is geodesically convex, the functional $F$ is very non-convex, but the DLSS equation shares the self-similar asymptotics of the heat equation, thanks to the intimate relation between $H$ and $F$. This talk is about a sixth order nonlinear PDE that is a gradient flow for the second-order functional $E(\rho) = \int \rho \| \nabla^2 \log \rho \|^2$. We prove existence of weak solutions, and then study their self-similar asymptotics using a structural relation connecting $E$ to both $H$ and $F$. 