

**Beyond treewidth: the tree-independence number**

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We introduce a new graph invariant called *tree-independence number*. This is a common generalization of treewidth and independence number, in the sense that bounded treewidth or bounded independence number implies bounded tree-independence number. The tree-independence number of a graph  $G$  is defined as the smallest positive integer  $k$  such that  $G$  has a tree decomposition whose bags induce subgraphs of  $G$  with independence number at most  $k$ . While for  $k = 1$  we obtain the well-known class of chordal graphs, we show that the problem of computing the tree-independence number of a graph is NP-hard in general.

We consider six graph containment relations (the subgraph, topological minor, and minor relations, as well as their induced variants) and for each of them completely characterize graph classes of bounded tree-independence number defined by a single forbidden graph with respect to the relation. In each of these bounded cases, a tree decomposition with small independence number can be computed efficiently, which implies polynomial-time solvability of the Maximum Weight Independent Set (MWIS) problem. This in particular applies to an infinite family of generalizations of the class of chordal graphs, for which a polynomial-time algorithm for the MWIS problem was given by Frank in 1976.