Removable singularities for anisotropic porous medium equations

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This paper is devoted to the obtaining conditions for removable singularity at the point for solutions of quasilinear parabolic equations model of which are

\[ u_t - \sum_{i=1}^{n} \left( u_{m_i-1} u_{x_i} \right)_{x_i} = 0, \]  
\[ u_t - \sum_{i=1}^{n} \left( u_{m_i-1} u_{x_i} \right)_{x_i} + f(u) = 0, \]  
\[ \frac{\partial u}{\partial t} - \sum_{i=1}^{n} \left( u_{m_i-1} u_{x_i} \right)_{x_i} + \sum_{i=1}^{n} |u_{x_i}|^{q_i} = 0, \]

We focus on the solutions which satisfy the initial condition

\[ u(x,0) = 0, \quad x \in \Omega \setminus \{0\}, \]  
where \( \Omega \) is a bounded domain in \( \mathbb{R}^n \), \( n \geq 2 \), \( t \in (0, T), 0 < T < +\infty, 0 \in \Omega \).

We suppose that the exponents \( m_i, q_i \), \( i = 1, n \) satisfy the following condition

\[ 1 - \frac{2}{n} < m_1 \leq m_2 \leq \ldots \leq m_n < m + \frac{2}{n}, \quad m = \frac{1}{n} \sum_{i=1}^{n} m_i, \]
\[ \frac{2 + nm}{1 + n} \leq q < 2, \quad \max_{0 \leq i \leq n} q_i < q \left( 1 + \frac{1}{n} \right), \quad \frac{1}{q} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{q_i}. \]

The main difficulty lies in the fact that part of \( m_i \) < 1 (singular case), and another part of \( m_i > 1 \) (degenerate case). We found a universal approach to study the properties of solutions of the anisotropic porous medium equation which not depends on the values of the anisotropic exponents \( m_i \). We established the pointwise condition for removability of the singularity for solutions of the equation (1) [1]. We also obtained the pointwise estimates of solutions, depending on the relations between the exponents \( m_i \) and \( q_i \) (for the equation (3) [3]), \( m_i \) and \( q \) (for the equation (2) in case \( f(u) = u^q \) [2]) which
guarantee that the point singularity is removable. The proof of removability result is based on the new a priori estimates of "large" type solutions. In particular, we obtain the Keller-Osserman type estimate of the solution to the problems (2), (4) and (3), (4).

References

