Decomposition formula for rough Volterra stochastic volatility models

Josep Vives[1]

joint work with R. Merino[1,4], J. Pospíšil[2], T. Sobotka[2], T. Sottinen[3]

[1]Facultat de Matemàtiques, Universitat de Barcelona, Spain
[2]NTIS-New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia, Czech Republic
[3]Department of Mathematics and Statistics, University of Vaasa, Finland
[4]VidaCaixa S.A., Investment Risk Management Department, Barcelona, Spain.

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Outline

Let $S = (S_t, t \in [0, T])$ be a strictly positive asset price process under a market chosen risk neutral probability measure $\mathcal{P}$ that follows the model:

$$\mathrm{d}S_t = rS_t\,\mathrm{d}t + \sigma_t S_t \left(\rho\,\mathrm{d}W_t + \sqrt{1-\rho^2}\,\mathrm{d}\tilde{W}_t\right), \tag{1}$$

where S_0 is the current price, $r \geq 0$ is the interest rate, W_t and $\tilde{W}_t$ are independent standard Wiener processes defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and $\rho \in (-1, 1)$.

Let $\mathcal{F}^W$ and $\mathcal{F}^{\tilde{W}}$ be the filtrations generated by W and $\tilde{W}$ respectively and let $\mathcal{F} := \mathcal{F}^W \vee \mathcal{F}^{\tilde{W}}$.

The volatility process σ_t is a square-integrable process assumed to be adapted to the filtration generated by W and its trajectories are assumed to be a.s. càdlàg and strictly positive a.e..

For convenience we let $X_t = \log S_t, t \in [0, T]$, and consider the model

$$\mathrm{d}X_t = \left(r - \frac{1}{2}\sigma_t^2\right) \mathrm{d}t + \sigma_t \left(\rho \, \mathrm{d}W_t + \sqrt{1-\rho^2} \, \mathrm{d}\tilde{W}_t\right). \tag{2}$$

Recall that $Z := \rho W + \sqrt{1-\rho^2}\tilde{W}$ is a standard Wiener process.

For any $t \in [0, T]$, $x \geq 0$ and $y \geq 0$ we denote by $BS(t, x, y)$ the so-called Black-Scholes function given by

$$BS\,(t, x, y) = e^x \Phi(d_+) - K e^{-r\tau} \Phi(d_-), \tag{3}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal law, $r \geq 0$ is a constant, $\tau = T - t$ and

$$d_\pm(y) = \frac{x - \ln K + (r \pm \frac{y^2}{2})\tau}{y\sqrt{\tau}}. \tag{4}$$

Recall that the price of an European plain vanilla call option under the classical Black-Scholes model with constant volatility σ, current log stock price X_t, time to maturity $\tau = T - t$, strike price K and interest rate r is given by $BS(t, X_t, \sigma)$.

In our setting, the call option price is given by

$$V_t = e^{-r\tau}\mathbb{E}_t[(e^{X_T} - K)^+] \tag{5}$$

where $\mathbb{E}_t$ is the conditional expectation respect to the $\sigma-$algebra $\mathcal{F}_t$.

Recall that from the Feynman-Kac formula for the model (2), the operator

$$\mathcal{L}_y := \partial_t + \frac{1}{2}y^2\partial_x^2 + \left(r - \frac{1}{2}y^2\right)\partial_x - r \tag{6}$$

satisfies $\mathcal{L}_y BS(t, x, y) = 0$, $t \in [0, T]$, $x \geq 0$ and $y \geq 0$.

It is well known that if the stochastic volatility process is independent of the price process, then the pricing formula of a plain vanilla European call is given by

$$V_t = \mathbb{E}_t[BS(t, S_t, \bar{\sigma}_t)]$$

where $\bar{\sigma}_t^2$ is the so called average future variance that is defined by

$$\bar{\sigma}_t^2 := \frac{1}{T-t} \int_t^T \sigma_s^2 \, \mathrm{d}s.$$

Naturally, $\bar{\sigma}_t$ is called the average future volatility.

We consider the adapted projection of the average future variance

$$v_t^2 := \mathbb{E}_t(\bar{\sigma}_t^2) = \frac{1}{T-t}\int_t^T \mathbb{E}_t[\sigma_s^2]\,\mathrm{d}s$$

Our goal is to obtain a decomposition of V_t in terms of v_t. This switches an anticipative problem related with the anticipative process $\bar{\sigma}$ into a non-anticipative one related to the adapted process v.

Consider the martingale $M_t = \int_0^T \mathbb{E}_t\left[\sigma_s^2\right]\,\mathrm{d}s$. We can write

$$dv_t^2 = \frac{1}{T-t}\left[dM_t + \left(v_t^2 - \sigma_t^2\right)\,\mathrm{d}t\right].$$

We define the operators $\Lambda := \partial_x$, $\Gamma := (\partial_x^2 - \partial_x)$ and $\Gamma^2 = \Gamma \circ \Gamma$.
In particular, for the function BS we get

$$
\begin{aligned}
\Gamma BS(t,x,y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right), \\
\Lambda\Gamma BS(t,x,y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right)\left(1 - \frac{d_+(y)}{y\sqrt{\tau}}\right), \\
\Gamma^2 BS(t,x,y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right)\frac{d_+^2(y) - yd_+(y)\sqrt{\tau} - 1}{y^2\tau}.
\end{aligned}
$$

The following lemma is also useful in proofs:

Lemma

Let $0 \leq t \leq T$. Then for every $n \geq 0$, there exists a constant C_n such that

$$|\Lambda^n \Gamma BS(t, X_t, v_t)| \leq \frac{C_n}{(\sqrt{T-t} \cdot v_t)^{n+1}}. \tag{7}$$

Extending the ideas of Alòs (2012) to jump diffusion models, Merino, Pospíšil, Sobotka, and Vives (2018) derived the following generic decomposition formula that is a consequence of the application of Itô and Feynman-Kac formulas.

Let

- B_t be a continuous semimartingale with respect to the filtration $\mathcal{F}^W$
- $A(t, x, y)$ be a $C^{1,2,2}([0, T] \times [0, \infty) \times [0, \infty))$ function
- v_t^2 and M_t be defined as above.
- $A_u := A(u, X_u, v_u^2)$

Theorem

$$
\begin{aligned}
& \mathbb{E}_t\left[e^{-r(T-t)}A_T B_T\right] = A_t B_t + \mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\partial_y A_u B_u \frac{1}{T-u}\left(v_u^2-\sigma_u^2\right)\mathrm{d}u\right] \\
+ \; & \mathbb{E}_t\left[\int_t^T e^{-r(u-t)} A_u \,\mathrm{d}B_u\right] + \frac{1}{2}\mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\left(\partial_x^2-\partial_x\right)A_u B_u\left(\sigma_u^2-v_u^2\right)\mathrm{d}u\right] \\
+ \; & \frac{1}{2}\mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\partial_y^2 A_u B_u \frac{1}{(T-u)^2}\,\mathrm{d}\langle M,M\rangle_u\right] \\
+ \; & \rho\mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\partial_{x,y}^2 A_u B_u \frac{\sigma_u}{T-u}\,\mathrm{d}\langle W,M\rangle_u\right] \\
+ \; & \rho\mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\partial_x A_u \sigma_u \,\mathrm{d}\langle W,B\rangle_u\right] + \mathbb{E}_t\left[\int_t^T e^{-r(u-t)}\partial_y A_u \frac{1}{T-u}\,\mathrm{d}\langle M,B\rangle_u\right].
\end{aligned}
$$

In our setting, for function BS, we have the following corollary:

Corollary

Let $0 \leq t \leq T$. Then,

$$V_t = BS(t, X_t, v_t) + \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \Gamma^2 BS(u, X_u, v_u) \, \mathrm{d}\langle M, M \rangle_u \right] + \frac{\rho}{2} \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \Lambda \Gamma BS(u, X_u, v_u) \sigma_u \, \mathrm{d}\langle W, M \rangle_u \right],$$

This formula is a consequence of the previous formula and the previous lemma applied to function BS.

The idea of the approximation is to write

$$V_t = BS(t, X_t, v_t) + \Gamma^2 BS(t, X_t, v_t) R_t + \Lambda\Gamma BS(t, X_t, v_t) U_t + \text{“error”},$$

where

$$R_t = \frac{1}{8}\mathbb{E}_t\left[\int_t^T \mathrm{d}\langle M, M\rangle_u\right] \quad \text{and} \quad U_t = \frac{\rho}{2}\mathbb{E}_t\left[\int_t^T \sigma_u \, \mathrm{d}\langle M, W\rangle_u\right]$$

where $\langle \cdot, \cdot \rangle$ denotes the quadratic covariation process.

- In case of Heston model, the terms R_t and U_t have nice analytic forms (Alòs, 2012), i.e. one can calculate them very quickly.
- Main goal: find R_t and U_t for rough Volterra SV models.

The error can be estimated applying the general decomposition another time. Using

$$A(t, X_t, v_t^2) = \Lambda\Gamma BS(t, X_t, v_t), \quad B_t = U_t \tag{8}$$

a decomposition of the term (I) can be found, and using

$$A(t, X_t, v_t^2) = \Gamma^2 BS(t, X_t, v_t), \quad B_t = R_t \tag{9}$$

a decomposition of the term (II) is obtained.

The terms of that appear on the new decompositions can be bounded and the error of the previous approximation pricing formula can be controlled.

We consider model (2) with a general Volterra volatility process defined as

$$\sigma_t := g(t, Y_t), \quad t \geq 0, \tag{10}$$

where $g : [0, +\infty) \times \mathbb{R} \mapsto [0, +\infty)$ is a deterministic function such that σ_t belongs to $L^1(\Omega \times [0, +\infty))$ and $Y = \{Y_t, t \geq 0\}$ is the Gaussian Volterra process

$$Y_t = \int_0^t K(t, s)\, \mathrm{d}W_s, \tag{11}$$

where $K(t, s)$ is a kernel such that for all $0 \leq s < t \leq T$,

$$\int_s^T K^2(t, s)\, \mathrm{d}t < \infty, \quad \int_0^t K^2(t, s)\, \mathrm{d}s < \infty, \tag{A1}$$

and

$$\mathcal{F}_t^Y = \mathcal{F}_t^W. \tag{A2}$$

Let $r(t,s)$ denote the autocovariance function of Y_t and $r(t)$ the variance. That is,

$$r(t,s) = \mathbb{E}[Y_t Y_s], \quad t,s \geq 0, \quad r(t) = r(t,t) = \mathbb{E}[Y_t^2], \quad t \geq 0. \tag{12}$$

Theorem (Prediction law for Gaussian Volterra processes (Sottinen and Viitasaari, 2017))

Let $\{Y_t, t \geq 0\}$ be the Gaussian Volterra process (11) satisfying assumptions (A1) and (A2). Then, the conditional process $(Y_u | \mathcal{F}_t, 0 \leq t \leq u)$ is Gaussian with $\mathcal{F}_t$-measurable mean function

$$\hat{m}_t(u) = \mathbb{E}_t[Y_u] = \int_0^t K(u,s)\, \mathrm{d}W_s$$

and deterministic covariance function (with $u_1, u_2 \geq t$)

$$\hat{r}(u_1, u_2 | t) = \mathbb{E}_t\left[\left(Y_{u_1} - \hat{m}_t(u_1)\right)\left(Y_{u_2} - \hat{m}_t(u_2)\right)\right] = r(u_1, u_2) - \int_0^t K(u_1, v) K(u_2, v)\, \mathrm{d}v.$$

Lemma (Auxiliary terms in the decomposition formula for the general volatility model)

Let $0 \leq t \leq u$ *and* $F(t, \hat{m}_t(u)) = \mathbb{E}_t(\sigma_u^2) = \mathbb{E}_t\left[g^2(u, Y_u)\right]$, *then*

$$\mathrm{d}F(t, \hat{m}_t(u)) = \left(\partial_1 F(t, \hat{m}_t(u)) + \frac{1}{2}\partial_{22} F(t, \hat{m}_t(u))\, K^2(u, t)\right) \mathrm{d}t + \partial_2 F(t, \hat{m}_t(u))\, \mathrm{d}\hat{m}_t(u). \tag{13}$$

$$\mathrm{d}\langle M, W\rangle_t = \int_0^T \partial_2 F(t, \hat{m}_t(u)) K(u, t)\, \mathrm{d}u\, \mathrm{d}t.$$

$$\mathrm{d}\langle M, M\rangle_t = \int_0^T \int_0^T \partial_2 F(t, \hat{m}_t(u_1)) \partial_2 F(t, \hat{m}_t(u_2)) K(u_1, t) K(u_2, t)\, \mathrm{d}u_1\, \mathrm{d}u_2\, \mathrm{d}t.$$

Assume now that X_t is the log-price process (2) with σ_t being the exponential Volterra volatility process

$$\sigma_t = g(t, Y_t) = \sigma_0 \exp\left\{\xi Y_t - \frac{1}{2}\alpha\xi^2 r(t)\right\}, \quad t \geq 0, \tag{14}$$

where $\{Y_t, t \geq 0\}$ is the Gaussian Volterra process (11) satisfying assumptions (A1) and (A2), $r(t)$ is its autocovariance function (12), and $\sigma_0 > 0$, $\xi > 0$ and $\alpha \in [0, 1]$ are model parameters.

Lemma (Auxiliary terms in the decomposition formula for the exponential Volterra volatility model)

Let σ_t *be as in* (14) *and* $0 \le t \le u$*. Then*

$$F(t, \hat{m}_t(u)) = \sigma_0^2 \exp\{2\xi \hat{m}_t(u) + 2\xi^2 \hat{r}(u|t) - \alpha\xi^2 r(u)\},$$

$$\mathrm{d}\langle M, W\rangle_t = 2\sigma_0^2 \xi \int_0^T \exp\left\{2\xi \hat{m}_t(u) + 2\xi^2 \hat{r}(u|t) - \alpha\xi^2 r(u)\right\} K(u,t)\, \mathrm{d}u\, \mathrm{d}t,$$

$$d\langle M, M\rangle_t = 4\sigma_0^4 \xi^2 \int_0^T \int_0^T \exp\{2\xi\left(\hat{m}_t(u_1) + \hat{m}_t(u_2)\right)\}$$

$$\cdot \exp\left\{2\xi^2\left(\hat{r}(u_1|t) + \hat{r}(u_2|t)\right)\right\} \cdot \exp\left\{-\alpha\xi^2\left(r(u_1) + r(u_2)\right)\right\} \cdot K(u_1,t)K(u_2,t)\, \mathrm{d}u_1\, \mathrm{d}u_2\, \mathrm{d}t.$$

Proposition (Terms in the approximation formula for the exponential Volterra volatility model)

Let σ_t be as in (14) and $0 \le t \le u$. Then we have detailed expressions for U_t and R_t. In particular,

$$U_0 = \rho\xi\sigma_0^3 \int_0^T \int_0^T e^{\alpha(s,u)} K(s,u)\, \mathrm{d}s\, \mathrm{d}u$$

with

$\alpha(s,u) = \frac{\xi^2}{2} \int_0^u [2K(s,z) + K(u,z)]^2 \,\mathrm{d}z + 2\xi^2 \hat{r}(s|u) - \frac{1}{2}\alpha\xi^2 r(u) - \alpha\xi^2 r(s)$ *and*

$$R_0 = \frac{1}{2}\sigma_0^4\xi^2 \int_0^T \int_0^T \int_0^T e^{\beta(s,u,v)} K(s,u)K(v,u)\,\mathrm{d}s\,\mathrm{d}v\, \mathrm{d}u$$

with

$\beta(s,u,v) = 2\xi^2 \int_0^u [K(s,z) + K(v,z)]^2 \,\mathrm{d}z + 2\xi^2 \left(\hat{r}(s|u) + \hat{r}(v|u)\right) - \alpha\xi^2 \left(r(s) + r(v)\right).$

Theorem (Upper error bound for the exponential Volterra volatility model)

Let X_t be a log-price process (2) with σ_t being the exponential Volterra volatility process (14). Then we can express the call option fair value V_t using the processes R_t, U_t from Proposition 7. In particular,

$$
\begin{aligned}
V_t = {} & BS(t, X_t, v_t) \\
& + \Lambda\Gamma BS(t, X_t, v_t) U_t \\
& + \Gamma^2 BS(t, X_t, v_t) R_t \\
& + \epsilon_t,
\end{aligned}
$$

where ϵ_t are error terms of order $O\left(\rho^2\xi + \xi^3\right)$.

Next step is to discuss possible elections for the kernel $K(t,s)$.

The most important example is the standard fractional Brownian motion (fBm) B_t^H :

$$B_t^H = \int_0^t K(t,s)\,\mathrm{d}W_s, \qquad (15)$$

where $K(t,s)$ is a kernel that depends on the Hurst parameter $H \in (0,1)$.

Recall that the covariance function of B_t^H is given by

$$r(t,s) := \mathbb{E}[B_t^H B_s^H] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right), \quad t,s \geq 0,$$

and in particular $r(t) := r(t,t) = t^{2H}$, $t \geq 0$.

As a first option, in αRFSV model we will assume that the volatility process

$$\sigma_t = \sigma_0 \exp\left\{\xi B_t^H - \frac{1}{2}\alpha\xi^2 r(t)\right\}, \quad t \geq 0, \tag{16}$$

where $\sigma_0 > 0$, $\xi > 0$ and $\alpha \in [0, 1]$ are model parameters together with empirical $H < 1/2$.

Note that for $\alpha = 0$ we have the classical exponential fractional volatility RFSV model and for $\alpha = 1$ we have the rBergomi model.

Since fBm is not a semimartingale, it is often useful to consider the so called approximated fractional Brownian motion (afBm), i.e. a process with Volterra kernel

$$\tilde{K}(t,s) = \sqrt{2H}(t-s+\varepsilon)^{H-1/2}\mathbf{1}_{\{s\le t\}}, \quad \varepsilon \ge 0, H \in (0,1). \tag{17}$$

For every $\varepsilon > 0$ such a process is a semimartingale and as ε tends to zero it converges to fBm in L^2 uniformly on $t \in [0,T]$. In this case

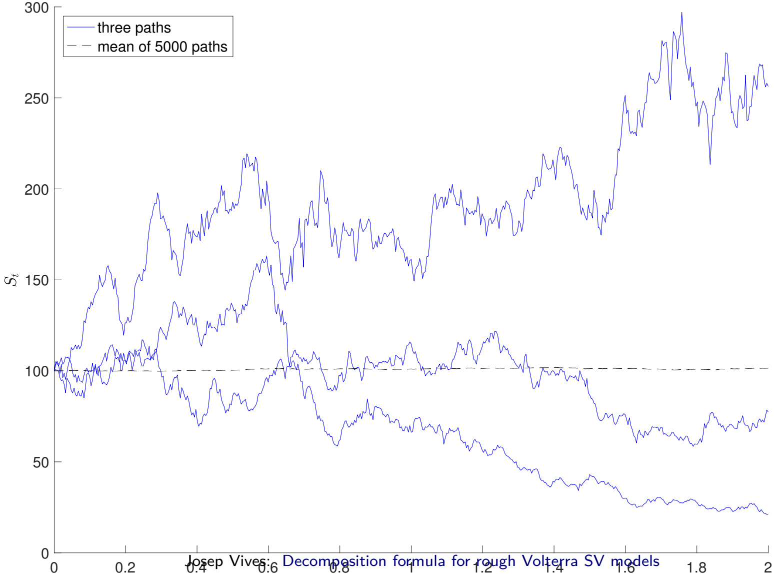
$$r(t) = \int_0^t \tilde{K}^2(t,v)\,\mathrm{d}v = 2H\int_0^t (t-v+\varepsilon)^{2H-1}\,\mathrm{d}v = (t+\varepsilon)^{2H} - \varepsilon^{2H},$$

$$\hat{r}(t|u) = r(t) - \int_0^u \tilde{K}^2(t,v)\,\mathrm{d}v = r(t) - 2H\int_0^u (t-v+\varepsilon)^{2H-1}\,\mathrm{d}v = (t-u+\varepsilon)^{2H} - \varepsilon^{2H}$$

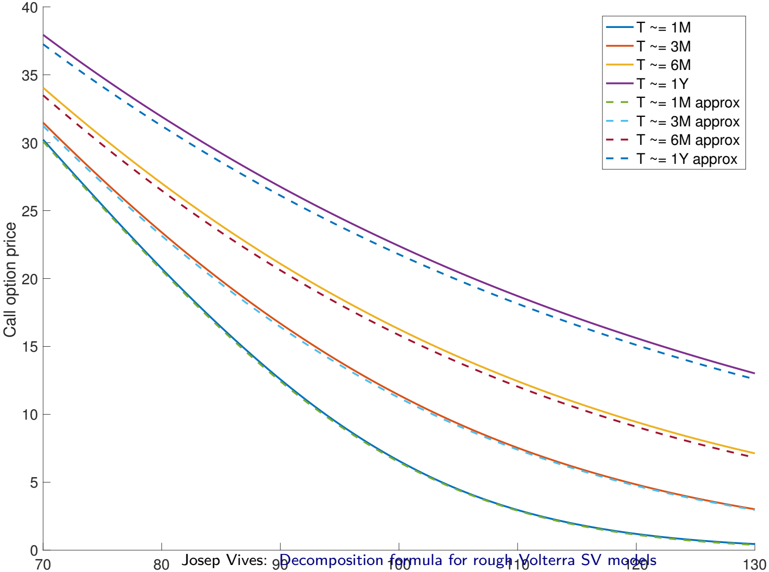
- For the *afBm* case we have also formulas for U_t and R_t.
- A particular case of the *afBm* is the case $\epsilon = 0$ and $H = \frac{1}{2}$, that is, the case of exponential standard Brownian motion. In this case, Y is also a semimartingale.
- In the αRFSV, if process Y is a semimartingale, the error of the approximation formula is of order $O\left(\rho^2\xi^2 + \xi^4\right)$
- It is clear that choosing other functions g and other kernels $K(t,s)$, new rough volatility models can be proposed and it is a matter of algebraic operations to obtain the corresponding approximation formula.

- McCrickerd and Pakkanen (2018) applied a composition of variance reduction methods to the simulation of rBergomi model (Turbocharging Monte Carlo simulation) based on the first order hybrid scheme (Bennedsen, Lunde, and Pakkanen, 2017) for the Volterra process and applying antithetic sampling, conditional MC and control variates to achieve the variance reduction (and a speed-up by a factor of ca 20).
- We have modified the code for the αRFSV model (2) with volatility process (16) and (17) with $\epsilon = 0$.
- For comparison purposes, we also simulated fBm using the Cholesky method with similar turbocharging techniques applied to it (parallel research).

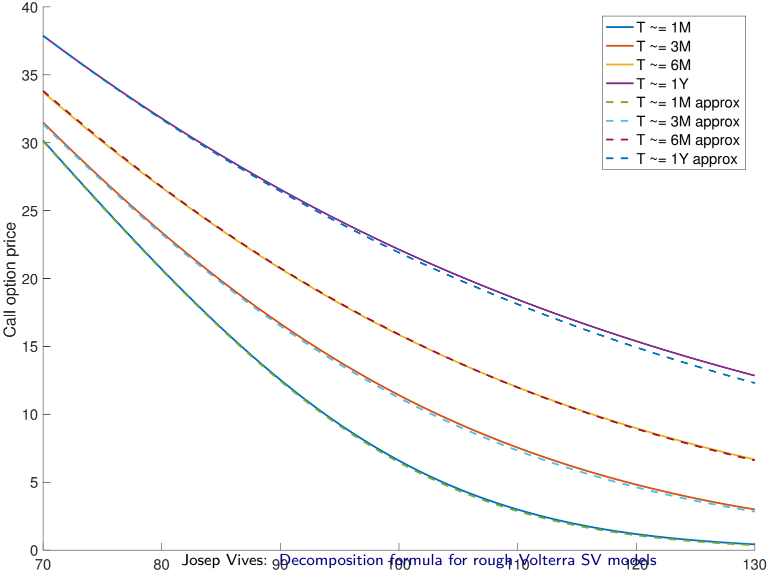
$H = 0.1, \alpha = 0, \rho = -0.2, \xi = 0.1, \sigma_0 = 0.3, S_0 = 100, r = 0.003, T = 2$



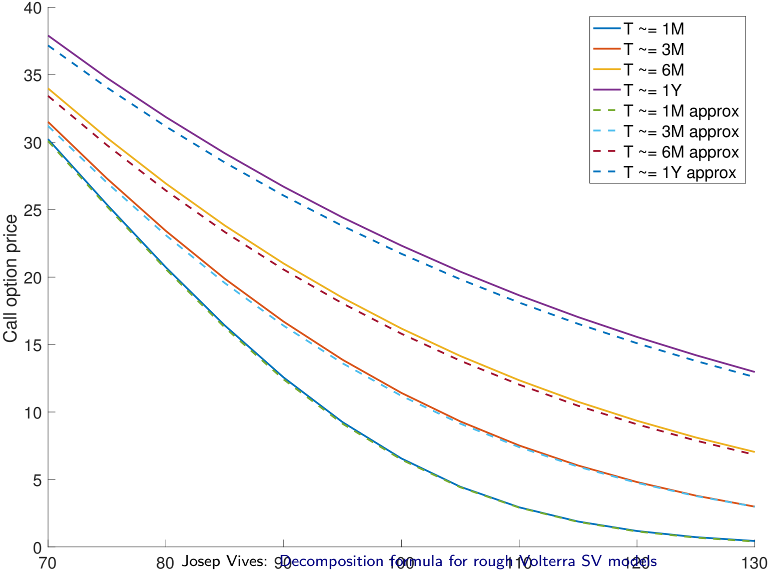
Example: $S_0 = 100, r = 0.003, \rho = -0.2, \sigma_0 = 0.3, \xi = 0.1, H = 0.1$.



Example: $S_0 = 100, r = 0.003, \rho = -0.2, \sigma_0 = 0.3, \xi = 0.1, H = 0.5$.



Example: $S_0 = 100, r = 0.003, \rho = -0.2, \sigma_0 = 0.3, \xi = 0.1, H = 0.1$.



Market data and calibration technique used

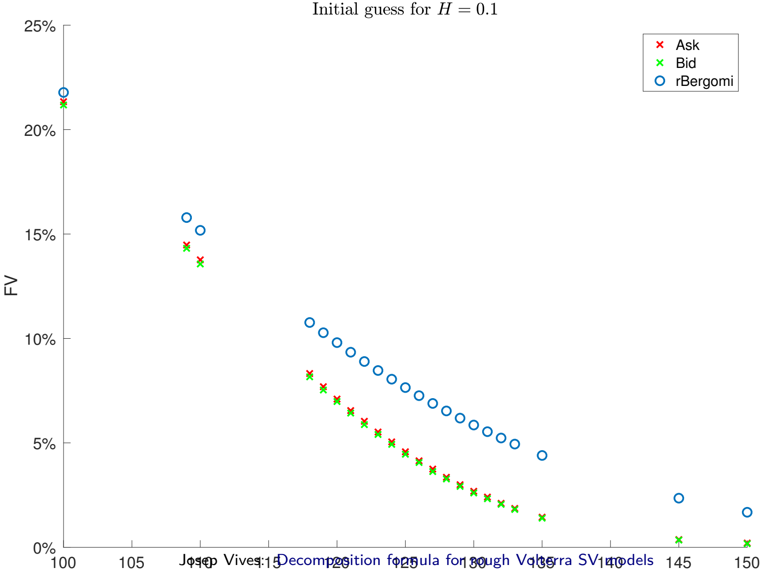
Market data:

- options on AAPL from 2015-04-15 (Bloomberg),
- 158 options in total, 1M-maturity: 22 options,
- we focus on short-maturity, where other models (such as Heston) behave poorly (Pospíšil, Sobotka, and Ziegler, 2019),
- spot $S_0 = 126.78$.

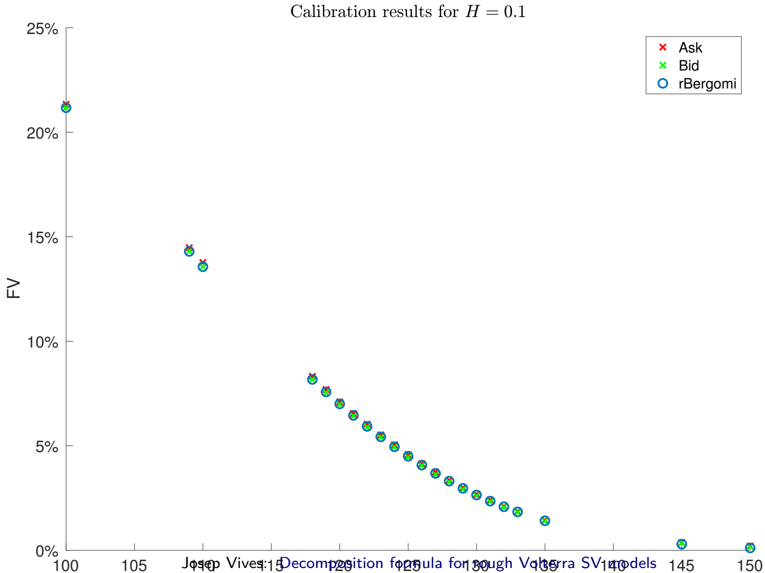
Optimization:

- unweighted nonlinear least squares (trust region reflective) for option prices - new approximation formula for rBergomi model is used,
- simple bounds: $\sigma_0, \xi \in [0, 3], \rho \in [-1, 1]$, value of H fixed (could be also optimized),
- measured error: relative fair value (FV).

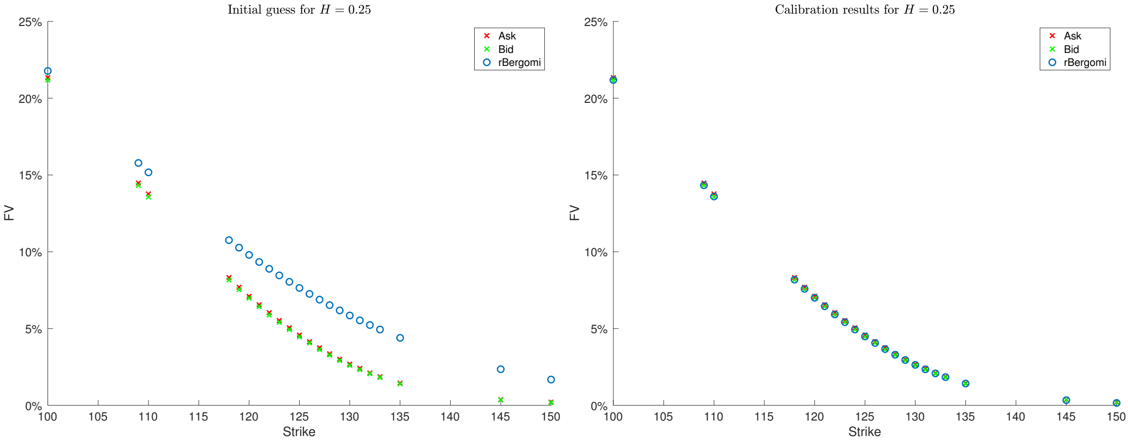
Initial guess: $\sigma_0 = 0.3, \xi = 0.1, \rho = -0.2$, fixed $H = 0.1$



Calibrated parameters: $\sigma_0 = 0.0861, \xi = 0.2245, \rho = -0.5944$



Calibrated parameters: $\sigma_0 = 0.0851, \xi = 0.5121 \rho = -0.2698$



Robustness and sensitivity analysis for αRFSV model is an ongoing research.

The newly obtained approximation formula was compared to the MC pricing approach introduced by Bennedsen, Lunde, and Pakkanen (2017) and McCrickerd and Pakkanen (2018). This enabled us to numerically verify the obtained solution, to quantify its approximation errors under various settings and, last but not least, to comment on suitability of the rBergomi model for calibration tasks to real market data based on AAPL stock options.

However, we remark that implementation of the approximation could be further improved – for simplicity we used a simple trapezoidal quadrature to numerically evaluate integrals appearing in U_t and R_t expressions.

The following conclusions were drawn:

1. The approximation error is well behaved for short maturities (typically for less than 1M) and the error increases with time to maturity and ξ parameter.
2. For medium-term expires we obtain well approximated prices only under low ξ regimes.
3. Although, the approximation under a rough Volterra process involves several numerical integration procedures, it is much faster than MC simulation approach implemented in the same environment.
4. Considering that the modeling approach studied has only few parameters, we were able to fit the sample market data surprisingly well. For calibration to short maturity smiles, we use just the approximation formula. For calibration to the whole surface, we use a new hybrid scheme which consist of a combination of approximation and simulation techniques. The idea is quite simple – to use approximation formula for low maturities or for low ξ values and MC simulations for the remaining computations. Suitability of this scheme was judged by a simple calibration to an ATMF-like backbone. We retrieved a well calibrated model, while saving a significant computational time compared to the calibration based on MC simulations only.

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Thank you for your attention!
Hvala!
Gràcies!