

Approaches of gifted pupils to solving algebraic word problems

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Abstract

Gifted students, and especially mathematically gifted students, differ from other students in their approach to solving mathematical problems. There are even tasks in which gifted pupils may be less successful than other pupils. The contribution will briefly discuss the issue of educating gifted students in mathematics and one algebraic word problem will illustrate how more complex thinking of gifted students can affect their success in the test task.

Introduction

Different approaches to defining the concept of “gift” which often differ significantly are found in literature. Various interpretations of this concept differ mainly in whether the gift is related to the potential of the individual or to their performance. Therefore, some approaches understand talent as a **manifestation** of excellent, above-average performance, others as the **potential** to deliver above-average performance in any valuable area. There is also the so-called IQ definition which refers to anyone who has an above-average value of the intelligence quotient, usually greater than or equal to 130, when the IQ of 100 is attributed to average individuals. The disadvantage of the IQ concept is its focus on the two components of intelligence, namely linguistic and mathematical-logical.

A pupil's gift is often inferred from their school results, most often the results of testing. A higher speed of problem solving is perceived as a manifestation of gift. However, gifted pupils can be slower in solving problems since they approach problem solving comprehensively. Their planning is called "global planning", as opposed to "local planning" used by less intelligent pupils. Therefore, gifted pupils do not have to prove their high ability when being tested. Mathematically gifted pupils may also encounter difficulties with their own complex way of thinking resulting in numerical mistakes. Moreover, some mathematically gifted pupils may be weak in arithmetic. For these reasons, testing may not be suitable for gifted pupils and their high abilities may not be reflected in it.

Research Methodology

The basic sample consisted of pupils of the 2nd grade of primary school and the corresponding years of multi-year grammar schools. The sample was chosen through availability sampling of 34 schools, with a total of 165 pupils. 28 primary schools, 1 primary school with extended mathematics teaching, 3 multi-year grammar schools, and 2 multi-year grammar schools with extended mathematics teaching participated in the research.

There were 109 boys and 56 girls in the sample. The sample consisted of 50 pupils of the 6th year, 31 pupils of the 7th year, 19 pupils of the 8th year and 65 pupils of the 9th year.

28 pupils were with mathematical talent, 55 pupils with general talent and "other" 82 above-average pupils participated in the research. There were only 13 pupils who had their talent recognized by the pedagogical-psychological counselling centre. Nine of these pupils had general talent and four were mathematically gifted. Pupils solved five different word problems that could be solved either arithmetically or algebraically. The results were analysed, and they are described in the study by Budínová (2018). Only one of the tasks will be listed here, for which it turned out that gifted pupils preferred more complex solutions and thus reduced their success.

Task

The rectangle in the picture is divided into three rectangles and a square. Determine the area of a square if the areas of three rectangles are known (in square centimetres). Write down the calculation.

18	27
54	

Results of the task

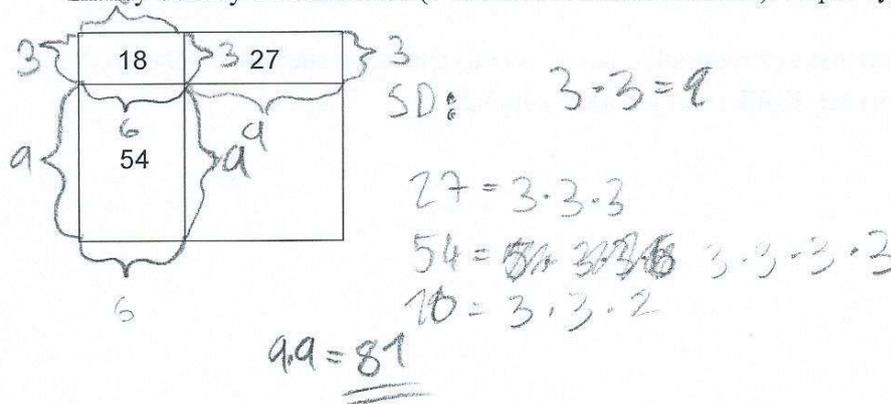
117 pupils, i.e., 71%, solved the task correctly. Success was strongly dependent on the choice of strategy. The most successful were pupils who chose an arithmetic strategy based on the search for common divisors. The 9th-year pupils who proceeded algebraically failed.

The resulting system of equations is not typical of school. The algebraic procedure was most often chosen by mathematically gifted pupils and pupils of a mathematical grammar school.

In this task, non-gifted pupils were more successful than mathematically gifted pupils. One of the reasons may be the fact that many mathematically gifted pupils tried to solve the problem algebraically, but they failed. The results point to the interesting fact that talent may not always lead to better test results.

The figure below illustrates a frequently used solution based on the search for common divisors of areas.

Obdélník na obrázku je rozdělen na tři obdélníky a čtverec. Urči obsah čtverce, jsou li známy obsahy tří obdélníků (v centimetrech čtverečních). Zapiš výpočet.



The following figure shows the algebraic solution of a pupil with Asperger's syndrome. He found an algebraic formula that could be modified to c^2 and in which he knew all the data. We can see a unique case of a pupil who is at the conceptual level of understanding algebraic procedures at the primary school. Obviously, his thinking is brilliant, but the notation is quite chaotic.

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Handwritten student solution showing the calculation of the area of a square inscribed in a rectangle. The student identifies the areas of three rectangles: $ab = 18$, $bc = 54$, and $ac = 27$. They use the formula $c^2 = \frac{ab \cdot bc}{ac}$ to find the area of the square, resulting in $c^2 = 81$.

This task was specific in that most pupils had never encountered a similar type of task. Therefore, we could observe the original approaches of the pupils who mostly did not know the necessary procedures from school. Most pupils (even the pupils of ordinary primary schools) appeared to be capable of mathematical deduction. This means that in many cases the pupils justified their decisions so that it would be clear that they were not only estimating the procedures, but they were also thinking in logical sequences.

Conclusions

Pupils' talent in mathematics is usually assessed by means of testing. However, the success of the solution often appears to be determined not only by the pupil's talent but also by other factors. Some pupils may fail in speed-focused testing (Budínová, 2018), especially gifted dyslectic pupils. Other gifted pupils have a complex way of thinking that slows down their solutions and can reduce their success. In the past, various researchers pointed out the controversy of testing in determining talent. The aim of this article was to examine the procedures and results of gifted and smart pupils while solving several tasks and to illustrate that in testing there are always more factors influencing success. The overall testing revealed that despite expectations, the gifted 9th year pupils were not always the most successful group. There were many reasons for this fact. When working with gifted pupils and creating math tests at school, we should be aware of the limits of testing and the factors that affect pupils' success in tests.

It is important to educate gifted pupils so that they could find a field of study in which they can achieve excellent results. In my experience, the gifted do not need a sticker, but optimal care that will allow them to develop their skills.

Otázka zazněla: Jak byli žáci vybíráni do výzkumu, na základě jakého testu? Takže je důležité je do výzkumu zapojit na základě nějakého standardizovaného testu, jinak se těžko obhájí, že matematicky nadaní dopadli hůře.

List of References

Budínová, I. (2018). *Přístupy nadaných žáků 1. a 2. stupně základní školy k řešení některých typů úloh v matematice*. Muni Press.

Sternberg, R. J. (1981). The evolution of theories of intelligence. *Intelligence*, 5(3), pp. 209-230.

Sternberg, R. J. & Williams, W., M. (2002). *Educational Psychology*. Allyn & Bacon.