Let $\mathbb{K}$ denote either the division algebra of quaternions $\mathbb{H}$ or that of octonions $\mathbb{O}$, and let $S \subset \mathbb{K}$ be the 2-sphere or, respectively, the 6-sphere of imaginary units, i.e. the sets of $I \in \mathbb{K}$ such that $I^2 = -1$. If $I \in \mathbb{K}$ we define the slice $C_I := \mathbb{R} + I \mathbb{R}$ and say that a domain $\Omega \subset \mathbb{K}$ is a slice domain if $\Omega \cap \mathbb{R} \neq \emptyset$ and $\Omega_I := \Omega \cap C_I$ is a domain in $C_I$ for any $I \in S$.

Let $\Omega \subset \mathbb{K}$ be a slice domain and let $f : \Omega \to \mathbb{K}$ be a function. If, for an imaginary unit $I$ of $\mathbb{K}$, the restriction $f_I := f|_{\Omega_I}$ has continuous partial derivatives and

$$\frac{\partial f(x + yI)}{\partial x} + I \frac{\partial f(x + yI)}{\partial y} = 0,$$

then $f_I$ is called holomorphic. If $f_I$ is holomorphic for all imaginary units of $\mathbb{K}$, then the function $f$ is called slice regular.

We refer the interested reader to the monograph [1] for an introduction to the main properties of slice regular functions in the quaternionic setting.

### Main results

As customary, a differentiable map will be called an immersion if its differential is injective at all points of the domain of definition.

Let $n, N$ be natural numbers with $N \geq n$ and let $\Omega$ be a domain in $\mathbb{R}^n$. An at least $C^1$ immersion $f : \Omega \to \mathbb{R}^N$ will be called a conformal or isothermal map if the matrix of the differential of $f$ is conformal, i.e., if it satisfies

$$df(p)df(p) = k(p)I_n$$

for a (never vanishing and at least $C^1$) function $k : \Omega \to \mathbb{R}$.

We specialize this definition for our purposes.

Let $\Omega$ be a slice domain in $\mathbb{H} \cong \mathbb{R}^4$ and let $N \geq 4$ be a natural number. Let $f : \Omega \to \mathbb{R}^N$ be an at least $C^1$ immersion. If, for any $I \in S$, $df_{C_I}$ and $df_{C_{I^*}}$ satisfy (2), then $f$ will be called a slice conformal or slice isothermal immersion.

### References
