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MY RECENT RESEARCH THEMES

My main interest has always been Group Theory.

I have been using insights from group theory to enlight questions coming from other parts of mathematics: rigidity of dynamical systems, differential geometry, number theory, and probability theory. Here is a list of eight independent exemples, roughly numbered by the year when they occured. This list illustrates the great variety of these applications.

Conformal dynamics (12).

With D. Hulin, we focus on the dynamic of the group G of conformal transformations of the sphere $\mathbb{S}^n$ acting on the space of non-singleton compact subsets of $\mathbb{S}^n$. We prove that the compact subsets whose G-orbit is closed are exactly the limit sets of convex cocompact subgroups of G. Surprisingly, this very nice description of conformally autosimilar fractals was not known.

Tempered reductive homogeneous spaces (13).

With T. Kobayashi, we give, for a real semisimple Lie group G and a closed connected subgroup H, a necessary and sufficient condition for the regular representation of G in $L^2(G/H)$ to be tempered, i.e. to be weakly contained in the regular representation in $L^2(G)$. Such a criterion is completely new even for symmetric spaces.

Central limit theorem in linear groups (14).

With J.F. Quint, we clarify the foundations of the theory of random products of matrices. Let μ be a probability measure on the linear group $\mathrm{GL}(d,\mathbb{R})$ whose action on $\mathbb{R}^d$ is strongly irreducible. We prove a central limit theorem for the corresponding random product of matrices when μ has a moment of order 2. This was only known, by works of LePage, Guivarch, and Raugi in the 80's, when μ has an exponential moment.

Spectral gap in compact Lie groups (15).

With N. de Saxcé, we study Sarnack's spectral gap conjecture which says: when μ is a symmetric probability measure on a compact simple Lie group whose support spans a dense subgroup, the convolution powers of μ equidistribute to the Haar measure at exponential speed. Generalizing Bourgain–Gamburd's result for $G = SU(d)$, we prove this conjecture when μ is supported by elements with algebraic coefficients.

Harmonic quasiisometric maps (16).

With D. Hulin, we prove that any quasiisometric map between complete Riemannian manifold with pinched negative curvature, is within bounded distance of a unique harmonic map. This solves a conjecture of Peter Li and Jiaping Wang from the 90's and extends previous work of Markovic for the real hyperbolic space.

Recurrence on the affine grassmannian (17).

With C. Bruère, we study the action of the affine group G of $\mathbb{R}^d$ on the space $X_{k,d}$ of k-dimensional affine subspaces. Given a compactly-supported Zariski dense probability measure μ on G, we show that $X_{k,d}$ supports a μ-stationary measure ν if and only if the $(k+1)^{\text{th}}$-Lyapunov exponent of μ is strictly negative. In particular, when μ is symmetric, ν exists if and only if $2k \geq d$. The only known case was when $k = 0$.

Totally geodesic planes in hyperbolic 3-manifolds (18).

With Hee Oh, we describe the closures of the orbits of a group H acting on an infinite volume quotient G/Γ when $H = SL(2, \mathbb{R})$, $G = SL(2, \mathbb{C})$ and Γ is an acylindrical geometrically finite discrete subgroup of G. The aim is to extend a very nice work of McMullen, Mohammadi and Oh from the convex cocompact case to the geometrically finite case.

Arithmeticity of discrete subgroups (19).

With S. Miquel, we prove that, in a higher rank simple Lie group, a discrete Zariski dense subgroup that contains an horospherical lattice is an arithmetic lattice of G. This solves a conjecture of Margulis from the 90's and extends previous works of Selberg and Hee Oh. This statement does not follow from the famous Margulis Arithmeticity theorem since we do not assume a priori that Γ has finite covolume.