CURRENT RESEARCH INTERESTS OF ALEXANDER EFIMOV

I am currently working on various problems and constructions related to triangulated categories in algebraic geometry and in mirror symmetry.

1) My recent paper on the categorical approach to the formal punctured neighborhood of infinity deals with the following problem. First, given a locally compact space (Hausdorff) space X, one can consider a complement $X - K$, where $K \subset X$ is a sufficiently large compact subset. More precisely, one should consider the formal inductive limit of such complements, where $K \subset X$ runs over all compact subsets. If we denote such an object by ∂X, then one can talk about local systems on ∂X, or about its homotopy type if X is sufficiently nice (e.g. locally finite CW complex).

In the algebro-geometric situation such an object is more complicated to construct, and even to define. There were two approaches: by Ben Bassat and Temkin in the framework of Berkovich spaces, and by Hennion, Porta and Vezzosi in the framework of derived stacks. My approach is to construct the derived category of coherent sheaves (equivalently, perfect complexes) on the formal punctured neighborhood of infinity, starting from a non-compact smooth algebraic variety. The construction does not depend on the choice of a compactification, and moreover it is defined for an arbitrary smooth differential graded category. It has been already applied by T. Pantev and B. Toën in studying moduli of flat bundles on non-compact varieties. The categorical nature of the construction also suggests applications to Fukaya categories.

2) I have a joint project with D. Auroux and L. Katzarkov on homological mirror symmetry for generalized Tate curves. The main result states that the derived category of the generalized Tate curve (of genus $g > 1$) is equivalent to the (suitably defined) Fukaya category of a certain trivalent configuration of two-dimensional spheres (considered as a singular symplectic manifold). Remarkably, the Schottky theta functions (certain summations over a free group on g generators) arise here on the both sides of mirror symmetry.

3) I have another paper in preparation on the K-theory of "large"categories, i.e. the (enhanced) triangulated categories with infinite direct sums. The usual algebraic K-theory of such categories vanishes by the standard Eilenberg swindle. I introduced a notion of a "continous algebraic K-theory"for a certain class of large categories, called dualizable categories. In particular, this can be applied to the derived categories of sheaves of modules on locally compact spaces. I computed the K-theory of such categories.

The particular special case is quite interesting, as it allows to somehow reduce the whole K-theory to the K_0 group: if F is an associative ring (or a DG ring), and $\mathrm{Sh}_F(\mathbb{R}^n)$ stands for the derived category of sheaves of F-modules on $\mathbb{R}^n$, then $K_0^{cont}(\mathrm{Sh}_F(\mathbb{R}^n)) = K_n(F)$.

A sketch of the definition of continous K-theory and proof of the main theorem is contained in the notes of my talk made by M. Hoyois http://www-bcf.usc.edu/ hoyois/papers/efimov.pdf